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Communication Systems

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Frequency modulation(FM)

Frequency modulation is that from of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal m(t)

$$f_i(t) = f_C + k_f m(t)$$

The term f_C represents the frequency of the unmodulated carrier and the constant k_f represents the **frequency sensitivity** of the modulator, expressed in hertz per volt on assumption that m(t) is a voltage waveform .

Integration equation with respect to time we get,(aftermultiplyingby2).

$$\theta_i(t) = 2\pi f_C + 2\pi k_f \int_0^t m(\tau) d\tau$$

The frequency modulated signal is therefore described in the time domain by

$s(t) = A_C \cos(\theta_i(t))$

$$s(t) = A_C \cos\left[2\pi f_C t + 2\pi k_f \int_0^t m(c)\,dc\right]$$

*In summary (see equations)

Angle modulated wave

$$s(t) = A_C \cos(2\pi f_c t + \phi(t))$$

Where

$$\phi(t) = \begin{cases} k_p m(t) & \leftarrow \text{PM} \\ 2\pi k_f \int_0^t m(c)\,dc & \text{FM} \end{cases}$$

$\leftarrow$

$\downarrow$

$$\frac{d\phi(t)}{dt} = \begin{cases} k_P \frac{d}{dt} m(t) & \leftarrow \text{PM} \\ 2\pi k_f m(t) & \leftarrow \text{FM} \end{cases}$$

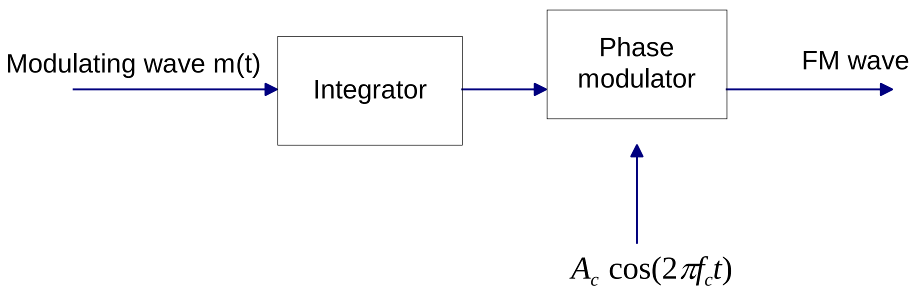
From equation it is clear that the envelope of a PM or FM signal is constant (equal to the carrier amplitude).

Where as the envelope of an AM signal is dependent on the message signal. Comparing equations reveals that an FM signal may be

regarded as a PM signal in which the modulating wave is :

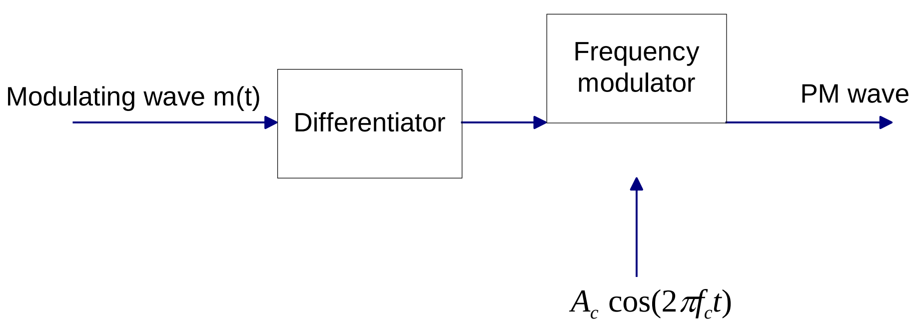
$\int_0^t m(\tau)\,d\tau$ in place of $m(t)$

This means that an FM signal can be generated by first $m(t)$ and the using integrating the result as the input to a phase modulator .



(Scheme for generating an FM wave)

Conversely, a PM signal can be generated by first differentiations m (t) and then using the result as the input to a frequency modulator as shown Fig .



(Scheme for generating a PM wave)

Scheme for generating PM wave

We may thus deduce all the properties of PM signal from those of FM signal or vice versa.
We concentrate our attention on FM signals.

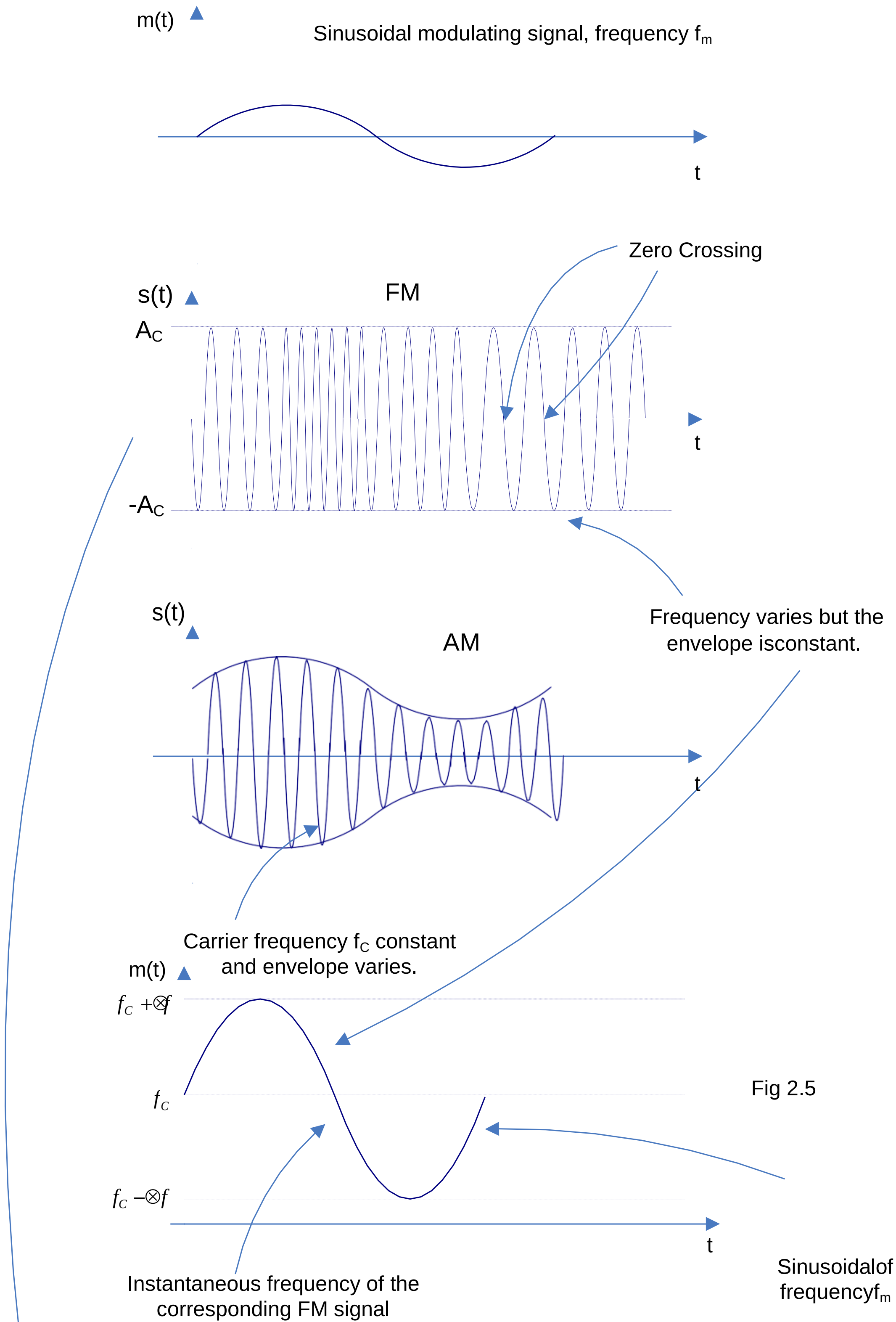


Fig 2.5

FM- message resides in the zero crossings of FM signal FM- wave does not look at all like the modulating waveform

The FM signals(t) defined by equations, $s(t)=A\cos\left[2\pi f_c t+2\pi k_f\int_0^t m(\tau)d\tau\right]$

Is a nonlinear function of the modulating signal m(t), which makes frequency modulation a nonlinear modulation process.

Consequently, unlike amplitude modulation, the **spectrum** of an FM signal is not related in a simple manner to that of a modulating signal –its analysing is much more different than that of an AM signal.

We propose two simple cases for the spectral analysis of an FM signal:

(1) A single tone modulation that produces a narrow FM signal.
(2) A single tone modulation that produces wideband FM signal.

Consider a sinusoidal modulating signal, $m(t)\ A_m$
$\cos(2 f_m\ )$
The instantaneous frequency of the resulting FM signal is given by:

$$f_i(t) = f_c + k_f m(t) \qquad \leftarrow \text{equation}$$

$$f_i(t) = f_c + k_f A_m \cos 2\pi f_m t$$

$$\boxed{f_i(t) = f_c + \otimes f \cos 2\pi f_m t}$$

The quantity $\otimes f = k_f A_m$ is called the **'frequency deviation'**, representing the **maximum departure** of the instantaneous frequency of the FM signal from the carrier frequency f_c.

$$\boxed{\otimes f = k_f A_m}$$

A fundamental characteristic of an FM signal is that the frequency deviation f is proportional to the amplitude of the modulating signal and is independent of the modulation frequency.

The angle $\theta_i(t)$ of the FMsignal,

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$$ (See 2.6)

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f A_m \frac{\sin \omega_m t}{\omega_m}$$

$$\theta_i(t) = 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$\theta_i(t) = 2\pi f_c t + \frac{\otimes f}{f_m} \sin(2\pi f_m t) \qquad (2.15).$$

The ratio of the frequency deviation f to the modulation frequency f_m is commonly called the **modulation index** of the FM signal.

$$\beta = \frac{\otimes f}{f_m} \qquad (2.16).$$

$$\Rightarrow \theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

β is measured in radians.

The FM signal is given by: $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ (2.17).

Depending on the value of the modulation index β, we may distinguish two cases of frequency modulation:

- **Narrowband FM,** for which β is small compared to one radian.
- **Wideband FM,** for which β is large compared to one radian.